SLT / file

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

SEPTEMBER 2021(2)

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat nie.*

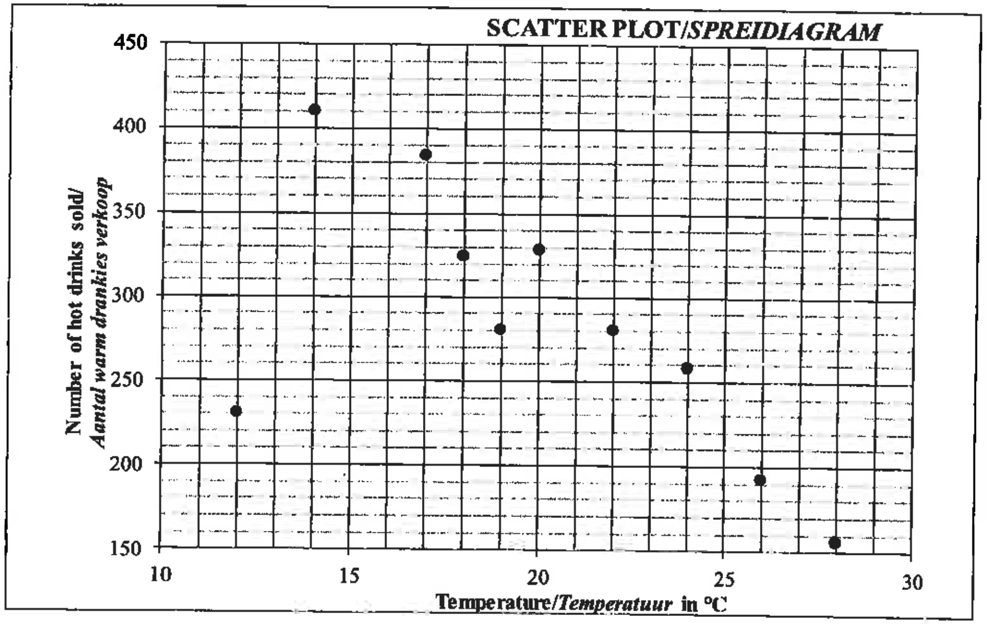
	GEOMETRY • *MEETKUNDE*
S	**A mark for a correct statement (A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	**A mark for the correct reason (A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

- If the c.v's are incorrect, you cannot award a not^n mark

cv ✓ not^n ✓

QUESTION/*VRAAG 1*

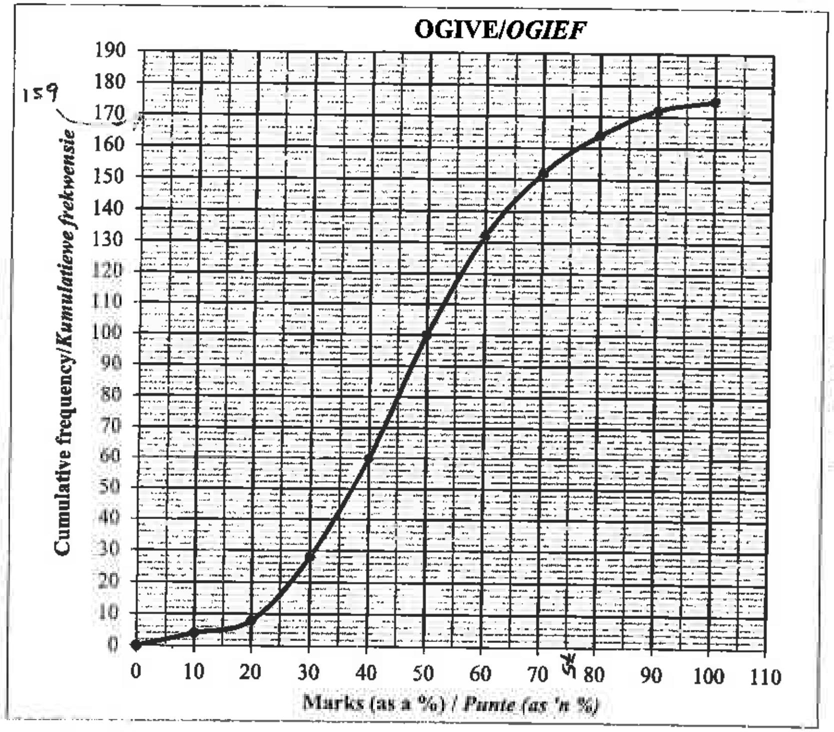
Temperature/ *Temperatuur* (in °C)	14	24	26	18	20	28	22	17	12	19
Number of hot drinks sold *Aantal warm drankies verkoop*	410	258	192	324	328	156	280	384	230	280



1.1	As the temperature increases the number of hot drinks sold decreases. / *Soos die temperatuur toeneem, neem die verkope van die warm drankies af.* **OR** As the temperature decreases the number of hot drinks sold increases. / *Soos die temperatuur afneem, neem die verkope van die warm drankies toe.*	✓ answer (1)	1
1.2	$a = 489{,}47$ ✓ $b = -10{,}37$ ✓ $\hat{y} = 489{,}47 - 10{,}37x$ ✓	✓ value of a ✓ value of b ✓ equation (3)	3

1.3	$\hat{y} = 489{,}47 - 10{,}37x$ $= 489{,}47 - 10{,}37(17)$ ✓ $= 313{,}18$ Number of hot drinks sold = 314 313 ✓313,18 Number of litres of milk $= \frac{314}{8}$ $\frac{313}{8}$ $\frac{313{,}18}{8}$ $= 39{,}25$ 39,125 39,1475 = 40 boxes of 1ℓ ✓	✓substitution ✓ 314 (accept 313) ✓ answer as N_0 (3)	3
1.4	The outlier is the point (12; 230).	✓(12; 230) (1)	1
		[8]	

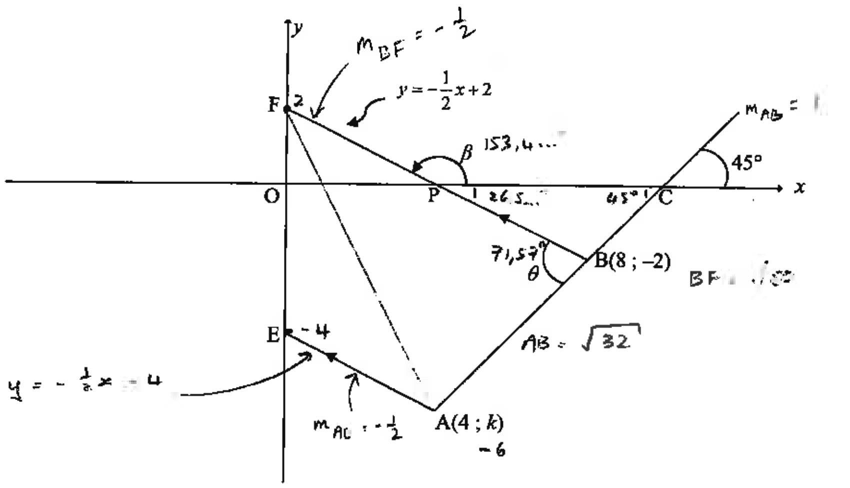
QUESTION/*VRAAG* 2



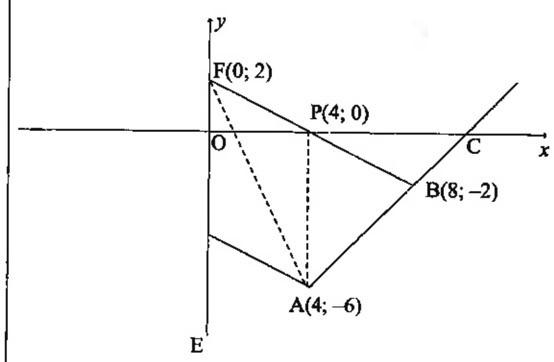
2.1.1	175	✓ answer (1)	1
2.1.2	$40 \le x < 50$ **OR** $40 < x \le 50$	✓ answer (1)	1
2.1.3	$175 - 158 = 17$ $175 - 159 = 16$	✓ 158 (accept 156 to 160) ✓ answer (accept 15 to 19) (2)	2
2.2.1	$\bar{x} = 74,87$	✓✓answer (2)	2
2.2.2	$\sigma = 16,12$	✓ answer (1)	1
2.2.3	$\bar{x} + \sigma = 74,87 + 16,12 = 90,99$ 3 learners	✓ 90,99 ✓ answer (2)	2

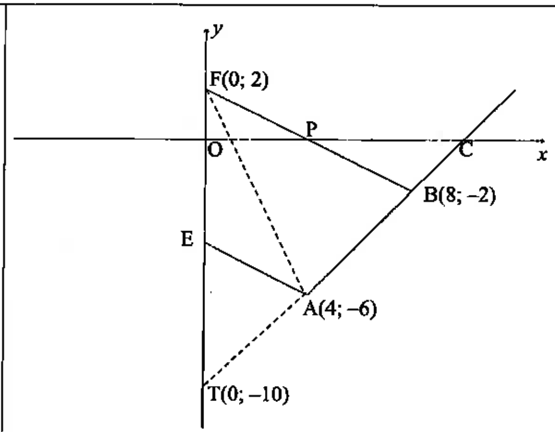
2.3	$\bar{x} - \sigma = 82,7$ $\bar{x} + \sigma = 94,1$ $2\bar{x} = 176,8$ $\bar{x} = 88,4$ $\sigma = 88,4 - 82,7$ OR $\sigma = 94,1 - 88,4$ $\sigma = 5,7$ $\quad\sigma = 5,7$ **OR** $\bar{x} = \frac{82,7 + 94,1}{2}$ $\bar{x} = 88,4$ $\sigma = 88,4 - 82,7$ OR $\sigma = 94,1 - 88,4$ $\sigma = 5,7$ $\quad\sigma = 5,7$	✓✓ $\bar{x} = 88,4$ ✓ answer (3) ✓✓ $\bar{x} = 88,4$ ✓ answer (3)	3
		[12]	

QUESTION/*VRAAG* 3



3.1	$m_{AB} = \tan 45° = 1$ ✓	✓ $m_{AB} = \tan 45° = 1$ (1)	1
3.2	$y = x + c$ $-2 = 8 + c$ $c = -10$ $y = x - 10$ ✓ eqn of AB $k = 4 - 10$ ✓ sub A (4 ; k) $k = -6$ given **OR** $\tan\theta = m_{AB}$ $1 = \frac{k-(-2)}{4-8}$ $\frac{k+2}{-4} = 1$ $k = -4 - 2$ $k = -6$	✓equation of AB ✓ substitute A in equation (2) ✓ substitute A & B into gradient formula ✓ equate to 1 (2)	2

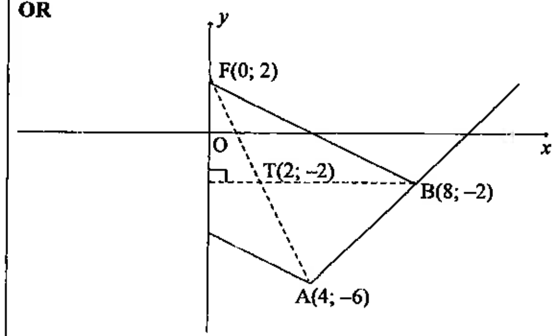
3.3	$m_{FB} = m_{EA} = -\frac{1}{2}$ [FB‖EA] $y = -\frac{1}{2}x + c$ ✓ $-6 = -\frac{1}{2}(4) + c$ ✓ $\therefore y = -\frac{1}{2}x - 4$ ✓ EA	✓ $m_{EA} = -\frac{1}{2}$ ✓ substitution of (4; −6) ✓equation (3)	3
3.4.1	$\tan\beta = -\frac{1}{2}$ ✓ $\beta = 153,4...°$ ✓ $\theta = 26,5...° + 45°$ [ext <of Δ] $= 71,57°$ ✓ S $\hat{P}_1 = 26,5...°$ ∠s on str line = 180° $\hat{C}_1 = 45°$	✓ $\tan\beta = -\frac{1}{2}$ ✓ value of β ✓ value of θ (3)	3
3.4.2	F(0;2) ✓ y int : y = 2 BF B(8;−2) $BF = \sqrt{(8-0)^2 + (-2-2)^2}$ ✓ $BF = \sqrt{80} = 4\sqrt{5}$ = 8,94 u ✓	✓F(0;2) ✓substitution ✓ answer (3)	3
3.4.3	 $0 = -\frac{1}{2}x + 2$ $x = 4$ $\therefore$ P(4; 0) $\therefore$ PA ‖ y-axis Area ΔABF = area ΔABP + area ΔAPF Area ΔABF $= \frac{1}{2}(6)(4) + \frac{1}{2}(6)(4)$ Area ΔABF = 24 units^2 **OR**	✓ P(4; 0) ✓ area of ΔABP ✓ area of ΔAPF ✓ answer (4)	



$y = x + c$
$-2 = 8 + c$
$c = -10$
$\therefore T(0; -10)$
Area ΔABF = area ΔFBT – area ΔAFT
Area $\Delta ABF = \frac{1}{2}(8)(12) - \frac{1}{2}(12)(4)$
Area ΔABF = 24 units^2

✓ C(0; –10)
✓ area of ΔABT
✓ area of ΔAFT
✓ answer
(4)

OR



$m_{AF} = \frac{-6-2}{4-0} = -2 \quad \therefore \ y = -2x + 2$
$-2 = -2x + 2$
$x = 2 \qquad \therefore T(2; -2)$
Area ΔABF = area ΔFTB + area ΔTBA
Area $\Delta ABF = \frac{1}{2}(6)(4) + \frac{1}{2}(6)(4)$
Area ΔABF = 24 units^2
OR

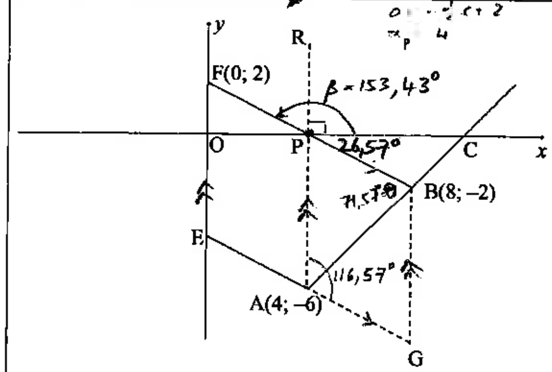
✓ T(2; –2)
✓ area of ΔFTB
✓ area of ΔTBA
✓ answer
(4)

Please turn over/*Blaai om asseblief*

3.4.3
A(4; –6)
B(8; –2)
$AB = \sqrt{(8-4)^2 + (-2-(-6))^2}$
$AB = \sqrt{32} = 4\sqrt{2}$ ✓ 5,656
Area of ABF $= \frac{1}{2}(AB)(BF)\sin A\hat{B}F$
$= \frac{1}{2}(\sqrt{32})(8,94)\sin 71,57°$ ✓ ✓ f s
$= 23,99$ u^2 ✓

✓ $AB = \sqrt{32} = 4\sqrt{2}$
✓ area formula
✓ substitution into area formula
✓ answer
(4) 4

3.5



RA ∥ y-axis
$C\hat{P}B = 26,57°$ ✓ s
$R\hat{P}B = 90° + 26,57°$ ✓
$R\hat{P}B = 116,57°$ ✓
PB ∥ AG
$\therefore P\hat{A}G = R\hat{P}B = 116,57°$ ✓ s [corresp ∠s; PB ∥ AG]

since $x_p = 4$
∠s on str line = 180°
knowing RA is vertical → $x_p = 4$

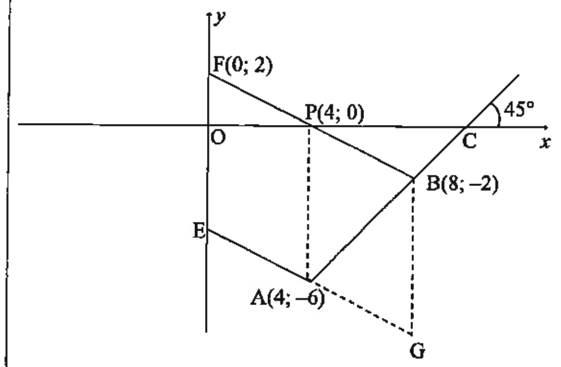
✓ $C\hat{P}B = 26,57°$
✓ $R\hat{P}B = 90° + C\hat{P}B$
✓ $R\hat{P}B$
✓ answer of $P\hat{A}G$
(4) 4

OR

$O\hat{F}P = 153,43° - 90°$ [ext ∠ of Δ]
$O\hat{F}P = 63,43°$
$F\hat{E}A = 180° - 63,43°$ [co-interior ∠ s; FB ∥ EA]
$= 116,57°$
$P\hat{A}G = 116,57°$ [corresp ∠ s; FE ∥ PA]

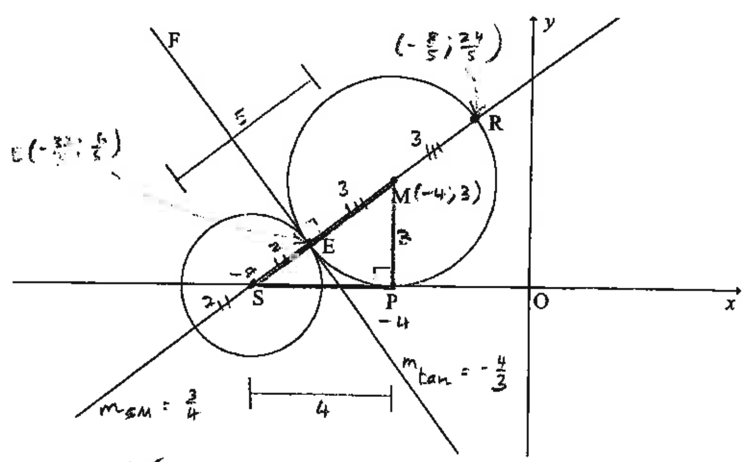
✓ $O\hat{F}P = 63,43°$
✓ $F\hat{E}A = 180° - 63,43°$
✓ $= 116,57°$
✓ answer of $P\hat{A}G$
(4)

Please turn over/*Blaai om asseblief*



	PA ∥ y-axis $P\hat{C}A = 45°$ [vert opp ∠s =] $P\hat{A}C = 45°$ [∠s of Δ] PA ∥ BG $B\hat{A}G = \theta = 71{,}57°$ [alt ∠s; PA ∥ BG] $P\hat{A}G = 45° + 71{,}57°$ $P\hat{A}G = 116{,}57°$	✓ $A\hat{P}C = 90°$ OR AP = PC ✓ $P\hat{A}C = 45°$ ✓ $B\hat{A}G = \theta = 71{,}57°$ ✓ answer of $P\hat{A}G$ (4)
		[20]

QUESTION/*VRAAG* 4



4.1.1	S(–8 ; 0)	✓ x-value ✓ y-value (2)	2
4.1.2	$r = 2$ ∴ diameter = 4 units	✓ $r = 2$ (1)	1
4.2.1	ER = 6 units EM = 3 units	✓ length of ER ✓ answer (2)	
4.2.2	$S(-8;0)\,;R\left(-\frac{8}{5};\frac{24}{5}\right)$ given $m_{SR} = \frac{0-\left(\frac{24}{5}\right)}{-8-\left(-\frac{8}{5}\right)}$ $= \frac{3}{4}$ $m_{FE} = \frac{-4}{3}$ [tan ⊥ rad] S	✓ substitution ✓ m_{SM} ✓ answer (3)	3
4.2.3	EM = MP = 3 units [radii] SM = 5 units $SP^2 = 5^2 - 3^2$ [Pythagoras] SP = 4 units ∴P(–4; 0) tan ⊥ rad ∴M(–4; 3)	✓ MP = 3 units ✓ length of SM ✓ length of SP ✓ coordinates of M (4)	4

* see alt. & notes ↓

• M not found in 4.2.3. ⇒ 0/2 for 4.2.4.

4.2.4	$\dfrac{x+\left(-\frac{8}{5}\right)}{2}=-4$ and $\dfrac{y+\frac{24}{5}}{2}=3$ $x=\dfrac{-32}{5}$ $y=\dfrac{6}{5}$ $\therefore E\left(\dfrac{-32}{5};\dfrac{6}{5}\right)$ $-6{,}4$ $1{,}2$ **OR** By translation: $E\left(\dfrac{-32}{5};\dfrac{6}{5}\right)$	✓x_E ✓y_E (2) ✓x_E ✓y_E (2)	2
4.3	$K(-5;-3)$ $SK=\sqrt{(-8-(-5))^2+(0-(-3))^2}$ ✓ $SK=\sqrt{18}$ ✓ $SK=3\sqrt{2}$ SK > 3 (radius of circle) ∴S lies outside the circle ✓ M(−4;3) (−5;3) refl x axis K(−5;−3)	✓ x-value ✓y-value ✓ substitution ✓ length of SK ✓ conclusion (5)	5
		[19]	

*

4.2.3 Let $M(x;y)$

$S(-8;0)$ SR

$SM = 5$

$\sqrt{(y-0)^2+(x-(-8))^2} = 5$

$y^2+(x+8)^2 = 25$ ✓

$y = \frac{3}{4}x + c$

sub $S(-8;0)$

$0 = \frac{3}{4}(-8)+c$

$6 = c$

$\therefore y = \frac{3}{4}x+6$ ✓

$\left(\frac{3}{4}x+6\right)^2 + (x+8)^2 = 25$

$\frac{9}{16}x^2 + 9x + 36 + x^2 + 16x + 64 = 25$

$\frac{25}{16}x^2 + 25x + 75 = 0$

$\div \frac{25}{16}$ $x^2 + 16x + 48 = 0$

$(x+4)(x+12) = 0$

$\therefore x = -4$ or $-\cancel{12}$ reject $< x_s$

$\therefore y = \frac{3}{4}(-4)+6$

$= 3$

$\therefore M(-4;3)$ ✓✓

4

4.2.3 * SR $\quad S\hat{P}M = 90°$ tan ⊥ rad

$y = \frac{3}{4}x + c$ $\quad$ NB ME = MP = 3 radii

Sub S(−8;0) $\quad \therefore y_M = 3$

$0 = \frac{3}{4}(-8) + c$

$6 = c$ $\quad \therefore 3 = \frac{3}{4}x + 6$

$\therefore y = \frac{3}{4}x + 6$ ✓ $\quad -3 = \frac{3}{4}x$

$-4 = x$

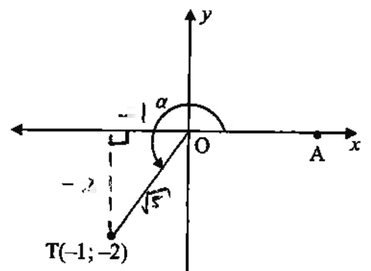
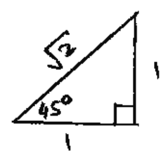
$\therefore M(-4; 3)$ ✓✓ $\quad$ 4

- if $y_M = 3$ is not fully justified
  - max ¼

and

- 0 for Q 4.2.4. because M is "given" later (in Q 4.3.)

QUESTION/*VRAAG* 5

5.1.1	 $\tan\alpha = \frac{-2}{-1} = 2$ ✓ $\quad \frac{y}{x}$ $\quad$ 1	✓answer (1)
5.1.2	$OT = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$ ✓5 Pythag $\cos\alpha = \frac{-1}{\sqrt{5}}$ ✓ $\quad \frac{x}{r}$ $\quad$ 2	✓ $OT = \sqrt{5}$ ✓ answer (2)
5.1.3	$\cos(\alpha + 45°)$ $= \cos\alpha\cos45° - \sin\alpha\sin45°$ $= \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{-\sqrt{2} + 2\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$ **OR** $\cos(\alpha + 45°)$ $= \cos\alpha\cos45° - \sin\alpha\sin45°$ ✓ $= \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ ✓ both $= \frac{-1+2}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ ✓ $\quad$ 4 	✓expansion ✓ substitution of sin α ✓ special angle ratios ✓answer (4) ✓expansion ✓ substitution of sin α ✓ special angle ratios ✓answer (4)

5.2	$2\sin(-20°).\sin160° - \cos40°$ $= 2(-\sin20°).\sin20° - \cos40°$ $= -2\sin^2 20° - (1 - 2\sin^2 20°)$ $= -1$ $\sin 160 = \sin(180°-20°) = +\sin 20°$	✓ $-\sin20°$ ✓ $\sin20°$ ✓ $1-2\sin^2 20°$ ✓answer (4)
5.3.1	$3\cos x.\sin x + \tan x.\cos^2(180° - x)$ $= 3\cos x.\sin x + \tan x.(-\cos x)^2$ $= 3\cos x.\sin x + \frac{\sin x}{\cos x}.\cos^2 x$ $= 4\cos x.\sin x$ $= 2\sin 2x$ $\cos^2(180°-x) = [\cos(180°-x)]^2 = [-\cos x]^2 = +\cos^2 x$ $\rightarrow 2(2\sin x\cos x)$	✓ reduction ✓ identity ✓ simplification ✓ single ratio (4)
5.3.2	$y \in [-2 ; 2]$ $-2 \le y \le 2$	✓critical values ✓ notation only if ev correct (2)
5.4	$\frac{\cos3x}{\cos x} = 4\cos^2 x - 3$ LHS $= \frac{\cos3x}{\cos x} = \frac{\cos(2x+x)}{\cos x}$ $= \frac{\cos2x\cos x - \sin2x\sin x}{\cos x}$ num $= \frac{(2\cos^2 x - 1)\cos x}{\cos x} - \frac{2\sin x\cos x\sin x}{\cos x}$ $= 2\cos^2 x - 1 - 2\sin^2 x$ $= 2\cos^2 x - 1 - 2(1 - \cos^2 x)$ $= 2\cos^2 x - 1 - 2 + 2\cos^2 x$ $= 4\cos^2 x - 3$ $=$RHS	✓ compound identity ✓ $2\cos^2 x - 1$ ✓ $2\sin x\cos x$ ✓ $1-\cos^2 x$ ✓expansion (5)
5.5	$3^{2\tan x} - 3^{\tan x + 1} = 54$ $3^{2\tan x} - 3.3^{\tan x} - 54 = 0$ $(3^{\tan x} - 9)(3^{\tan x} + 6) = 0$ $3^{\tan x} = 3^2$ or $3^{\tan x} = -6$ no solution $\tan x = 2$ $\therefore x = 63{,}43° + k.180°; k \in Z$ $k = 3^{\tan x}$ $k^2 - 3k - 54 = 0$ $(k-9)(k+6) = 0$ $k \in Z$ must be shown **OR** $\therefore x = 63{,}43° + k.360°; k \in Z$ or $x = 243{,}43° + k.360°; k \in Z$	✓ standard from ✓ factors ✓ both equations ✓ $\tan x = 2$ ✓ $x = 63{,}43° + k.180°; k \in Z$ (5) **OR** ✓ $x = 63{,}43° + k.360°; k \in Z$ & $243{,}43° + k.360°; k \in Z$ (5)
		[27]

4
4
2
5
NOT QE 0/5
5

5.4 $\frac{\cos 3x}{\cos x} = 4\cos^2 x - 3$

LHS $= \frac{\cos(2x+x)}{\cos x}$

$= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$

$= \frac{(2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x}{\cos x}$

$= \frac{2\cos^3 x - \cos x - 2\sin^2 x \cos x}{\cos x}$

$= \frac{2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x}{\cos x}$

$= \frac{2\cos^3 x - \cos x - 2(\cos x - \cos^3 x)}{\cos x}$

$= \frac{2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x}{\cos x}$

$= \frac{4\cos^3 x - 3\cos x}{\cos x}$

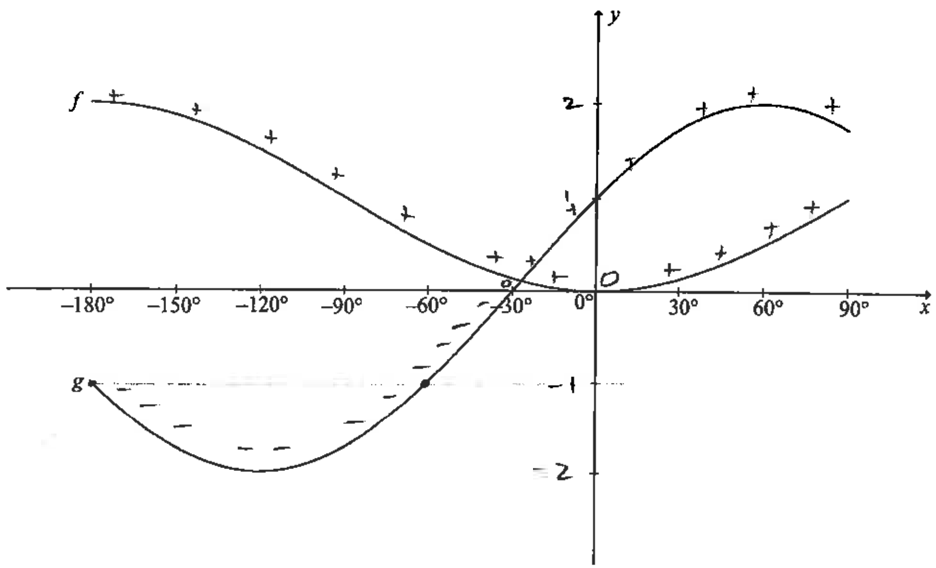
$= \frac{\cos x(4\cos^2 x - 3)}{\cos x}$ OR $\frac{4\cos^3 x}{\cos x} - \frac{3\cos x}{\cos x}$

$= 4\cos^2 x - 3$

$=$ RHS

5

QUESTION/*VRAAG* 6

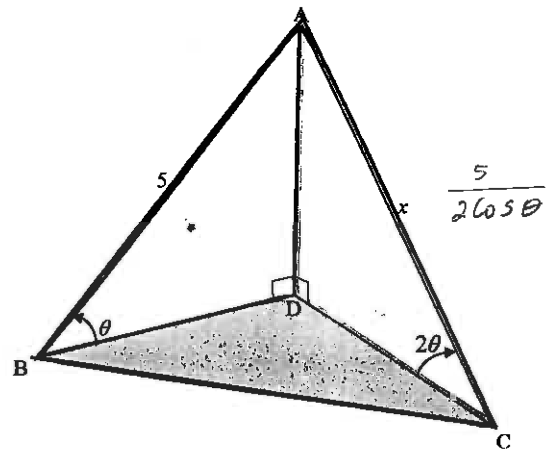


6.1.1	$x \in [-30°;90°]$ ✓✓ $f(x).g(x) \geq 0$ $y_f . y_g$	✓endpoints ✓notation (2)
6.1.2	$x = -180°$ or $-60°$ ✓ ✓ $g(x) = -1$ $y_g = -1$	✓ –180° ✓ –60° (2)
6.2	$f(x) = -\cos(x+90°)+1$ ✓ $= \sin x + 1$ ✓	✓ $\cos(x+90°)$ ✓ answer (2)
		[6]

$f(x) = -\cos(x+90°)+1$
$= -\cos(90°+x)+1$
$= -[-\sin x]+1$
$= \sin x + 1$

2
2

QUESTION/*VRAAG* 7

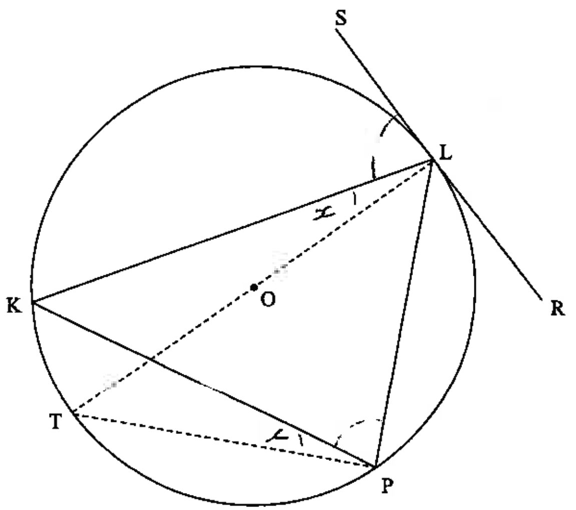


7.1	$\sin\theta = \frac{AD}{5}$ ✓ ΔABD $\frac{AD}{\sin\theta} = \frac{5}{\sin 90°}$ $AD = 5\sin\theta$ $\sin 2\theta = \frac{AD}{x}$ ✓ $\frac{AD}{\sin 2\theta} = \frac{x}{\sin 90°}$ $AD = x\sin 2\theta$ $= x.2\sin\theta\cos\theta$ ✓	✓ trig ratio ✓ trig ratio ✓ $2\sin\theta\cos\theta$
	$x.2\sin\theta\cos\theta = 5\sin\theta$ ✓ $x = \frac{5\sin\theta}{2\sin\theta\cos\theta}$ ✓ $= \frac{5}{2\cos\theta}$ given	✓ equating AD ✓ x as subject (5)
7.2	$BC^2 = 5^2 + \left(\frac{5}{2\cos 30°}\right)^2 - 2(5)\left(\frac{5}{2\cos 30°}\right)\cos 112°$ ✓ ✓ $= 44,147...$ ΔABC $BC = 6,64$ units ✓ $\frac{5}{2\cos 30°} = \frac{5\sqrt{3}}{3} = 2,88...$	✓ use ~~area~~ cos-rule correctly ✓ substitution ✓answer (3)
		[8]

5
3

QUESTION/*VRAAG* 8

8.1

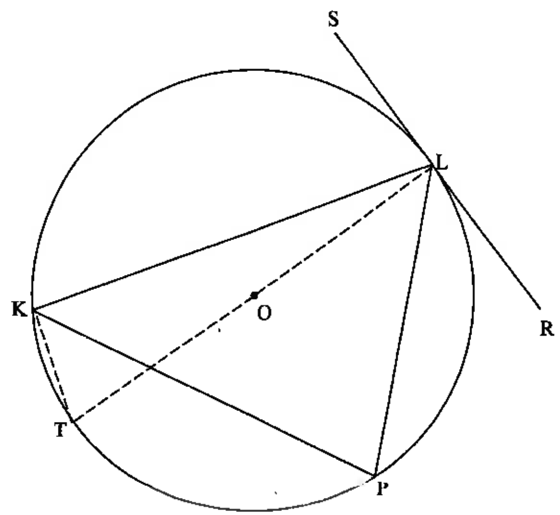


✓ Constr

8.1	Construction: Draw diameter LT and draw TP *Konstruksie: Trek middellyn* LT *en verbind* TP $\hat{SLK} = 90° - \hat{TLK}$ [radius ⊥ tangent/*raaklyn*] $\hat{TPL} = 90°$ [∠ in semi-circle/*semi-sirkel*] $\therefore \hat{KPL} = \hat{P} = 90° - \hat{TPK}$ $= 90° - \hat{TLK}$ [∠s same segment/∠*e dieselfde segment*] $\therefore \hat{SLK} = \hat{P}$	✓Constr ✓S ✓R ✓S /R ✓S ✓R (6)

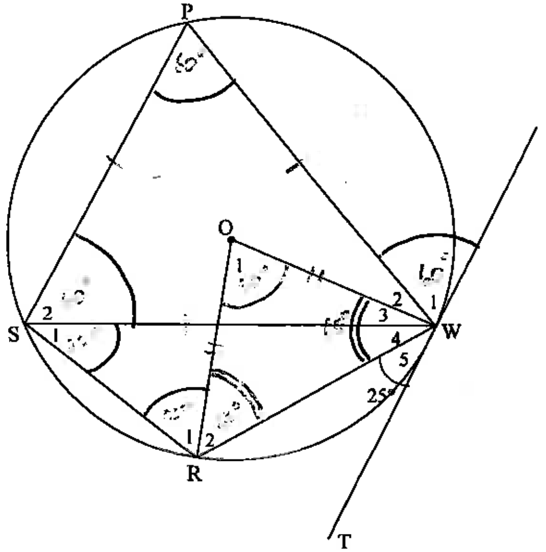
Let $\hat{L}_1 = x$
$\hat{SLK} + x = 90°$ ✓R tan ⊥ rad
$\therefore \hat{SLK} = 90° - x$ ✓S
$P_1 = x$ ✓S ✓R ∠'s in same ⊙ segm =
$\hat{LPK} + x = 90°$ ∠ in semi ⊙ = 90°
$\therefore \hat{LPK} = 90° - x$ ✓SR
$\therefore \hat{SLK} = \hat{LPK}$ both $= 90° - x$
" $\hat{P}$ "

6



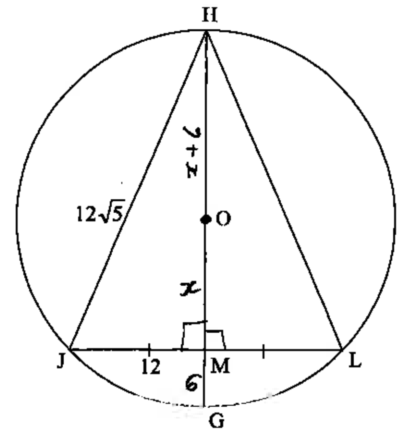
8.1	Construction: Draw diameter LT and draw KT *Konstruksie: Trek middellyn LT en verbind KT* $\hat{SLK} = 90° - \hat{TLK}$ [radius ⊥ tangent/*raaklyn*] $\hat{LKT} = 90°$ [∠ in half circle/*semi-sirkel*] $\therefore \hat{P} = \hat{KTL}$ [∠s same segment/∠*e dieselfde segment*] $= 90° - \hat{TLK}$ $\therefore \hat{SLK} = \hat{P}$	✓ construction ✓ S /R ✓ S✓ R ✓ S ✓ S / R (6)

8.2



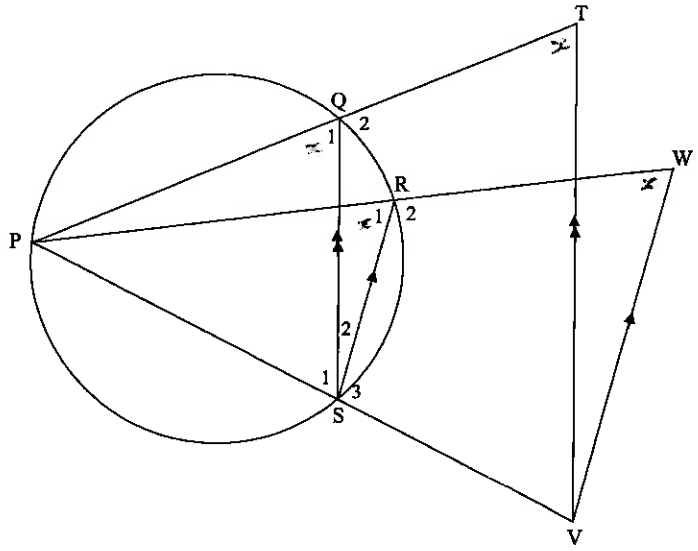
8.2.1(a)	$\hat{S}_1 = 25°$ ✓ ✓ [tan chord theorem/ ∠ *tussen raaklyn en koord*]	✓S ✓R (2)	2
8.2.1(b)	$\hat{O}_1 = 50°$ ✓ ✓[∠ at centre = 2×∠ at circumference / *midpts.* ∠ = 2 × *omtreks* ∠]	✓S ✓R (2)	2
8.2.1(c)	$\hat{R}_2 = \hat{W}_3 + \hat{W}_4 = 65°$ ✓radii, ∠s opp = sides, sum ∠s in Δ=180° $\hat{P} = 60°$ ✓S/R ∠s of equilat Δ $\hat{R}_1 = 55°$ ✓S ✓R [opp ∠ of cyclic quad / teenoorst. ∠*e van kvh*] 180° − (60° + 65°)	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)	
8.2.2	$\hat{W}_1 = \hat{S}_2 = 60°$ ✓S/R tan chord theorem, ∠s of equilat Δ $\hat{P} = 60°$ (8 2.1.) $\therefore \hat{W}_1 = \hat{P}$ ✓S both = 60° SP ∥ TW ✓R [alt ∠s = / *verwisselende* ∠*e gelyk*]	✓ S / R ✓ S ✓ R (3)	

8.3



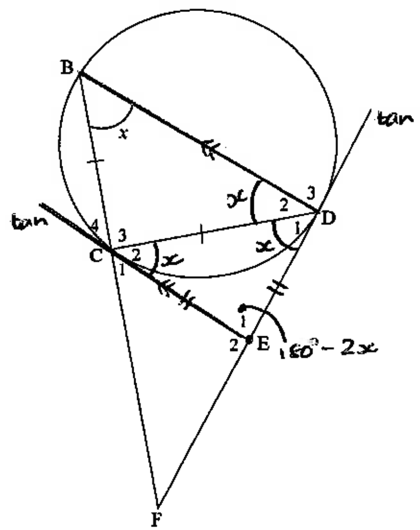
8.3.1	$OG = x + 6$ ✓ $\therefore HM = 2x + 6$ ✓	✓S ✓S (2)	
8.3.2	OM ⊥ JL [line from centre to midp of chord/*lyn van midpt halv kd*] $OJ^2 = JM^2 + OM^2$ [Pythagoras] $(x+6)^2 = 12^2 + x^2$ $x^2 + 12x + 36 = 144 + x^2$ $x = 9$ $r = 15$ units **OR** OM ⊥ JL ✓S ✓R [line from centre to midp of chord/*lyn van midpt halv kd*] $HJ^2 = HM^2 + JM^2$ [Pythagoras] $(12\sqrt{5})^2 = (2x+6)^2 + 12^2$ ✓ $720 = 4x^2 + 24x + 36 + 144$ $0 = 4x^2 + 24x - 540$ $0 = x^2 + 6x - 135$ $0 = (x-9)(x+15)$ $x = 9$ ✓ or $x \neq -15$ reject $r = 15$ units ✓	✓ S ✓R ✓ subst into Pyth ✓ value of x ✓ length of radius (5) ✓ S ✓R ✓ subst into Pyth ✓ value of x ✓ radius (5)	5
		[25]	

QUESTION/*VRAAG 9*



9.1	$\frac{TQ}{QP}=\frac{VS}{SP}$ ✓✓ [line ∥ 1 side of Δ] $\frac{VS}{SP}=\frac{WR}{RP}$ ✓ [line ∥ 1 side of Δ] $\therefore \frac{TQ}{QP}=\frac{WR}{RP}$ both = $\frac{VS}{SP}$	✓S ✓R ✓ S/R (3)	3
9.2.	Let $\hat{Q}_1 = x$ $\therefore \hat{R}_1 = x$ ✓✓ ∠s in the same segment $\therefore W = x$ ✓ corres ∠s, RS ∥ VW $\therefore \hat{Q}_1 = \hat{W}$ both x $\hat{Q}_1 = \hat{T}$ ✓[corres ∠s ,TV ∥ QS / *ooreenkomstige ∠e,* TV ∥ QS] $\therefore \hat{T} = \hat{W}$ both x ∴ TPVW is a cyclic quad [converse ∠s in the same segment / *lyn onderspan gelyke hoeke*] ✓	✓S ✓R ✓S/R ✓S ✓R (5)	5
		[8]	

QUESTION/*VRAAG 10*



10.1.1	$\hat{D}_1 = x$ ✓✓ [tan chord theorem / ∠ *tussen en raaklyn koord*] $\hat{C}_2 = \hat{D}_1 = x$ ✓✓ Tans from common pt. , ∠'s opp = sides $\hat{E}_1 = 180° - 2x$ ✓ [sum of int ∠s Δ; ∠e Δ] **OR** $\hat{D}_1 = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{C}_2 = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{E}_1 = 180° - 2x$ [sum of int ∠s Δ; ∠e Δ]	✓S ✓R ✓S✓ R ✓R (5) ✓S ✓R ✓S✓ R ✓R (5)	5
10.1.2	In ΔECD and ΔCBD Δ's E_1 C_2 D_1, C_3 & D_2 $\hat{C}_2 = \hat{B} = x$ ✓ [tan chord theorem / *raaklyn koordst.*] $\hat{D}_2 = \hat{B} = x$ ∠s opp equal sides $\therefore \hat{D}_1 = \hat{D}_2 = x$ ✓S ∴ ΔECD ∥∣ ΔCBD [A A A] ✓R **OR** In ΔECD and ΔCBD $\hat{C}_2 = \hat{B} = x$ [tan chord theorem / *raaklyn koordst.*] $\hat{D}_2 = \hat{B} = x$ [∠s opp equal sides / ∠ *teenoor gelyke sye*] $\therefore \hat{D}_1 = \hat{D}_2 = x$ $\hat{E}_1 = \hat{C}_3$ [3rd ∠ of Δ / ∠e Δ] ∴ ΔECD ∥∣ ΔCBD	✓S / R ✓S ✓R (3) ✓S / R ✓S ✓S (3)	3

10.2.1	$\frac{EC}{BC}=\frac{CD}{BD}=\frac{ED}{CD}$ [ΔECD ||| ΔCBD] $\frac{CD}{BD}=\frac{ED}{CD}$ ✓S $\Delta E\hat{C}D \parallel\!\mid \Delta C\hat{B}D$ $CD^2 = ED.BD$ ✓S $ED = CE$ ✓S tan's from ext common pt = / 10.1.1. $\therefore CD^2 = CE.BD$	✓S ✓$CD^2 = ED.BD$ ✓$ED = CE$ (3)
10.2.2	$\hat{C}_2 = \hat{D}_2 = x$ [proven / *reeds bewys*] BD || CE [alt ∠s = / *verwisselende* ∠ *gelyk*] $\therefore \frac{FE}{DE}=\frac{FC}{CB}$ [line || one side of Δ / *lyn* || *een sy van* Δ] $\therefore \frac{CF^2}{EF^2}=\frac{CB^2}{DE^2}$ $\therefore \frac{CF^2}{EF^2}=\frac{DE.BD}{DE^2}$ [CB = CD] $\therefore \frac{CF^2}{EF^2}=\frac{BD}{DE}$	✓S ✓R ✓S ✓R ✓ squaring ✓ subst $CD^2 = ED.BD$ (6)
		[17]

TOTAL/*TOTAAL*: 150

3

10.2.2. $\hat{C}_2 = \hat{D}_2$ both $= x$

$\therefore$ BD // CE ✓S alt ∠'s = ✓R

$\frac{FC}{CB} = \frac{FE}{ED}$ ✓S ✓R line // 1 side of Δ

$FC.ED = FE.CB$

$\frac{CF}{EF} = \frac{CB}{DE}$

$\frac{CF}{EF} = \frac{CD}{DE}$ given CB = CD

$\frac{CF^2}{EF^2} = \frac{CD^2}{DE^2}$ ( )² both sides ✓

$= \frac{CE.BD}{DE.DE}$ ✓ sub 10.2.1.

$= \frac{DE.BD}{DE.DE}$ 10.1.1. / 10.2.1. CE = DE

$= \frac{BD}{DE}$

6

10.2.1 $\frac{CD}{BD} = \frac{ED}{CD}$ ✓ $\Delta E\hat{C}D \parallel\!\mid \Delta C\hat{B}D$

$CD^2 = ED.BD$ ✓

but $ED = CE$ ✓ tan's from ext common pt =

$\therefore CD^2 = CE.BD$

OR

$\frac{EC}{CB} = \frac{CD}{BD}$ ✓ $\Delta E\hat{C}D \parallel\!\mid \Delta C\hat{B}D$

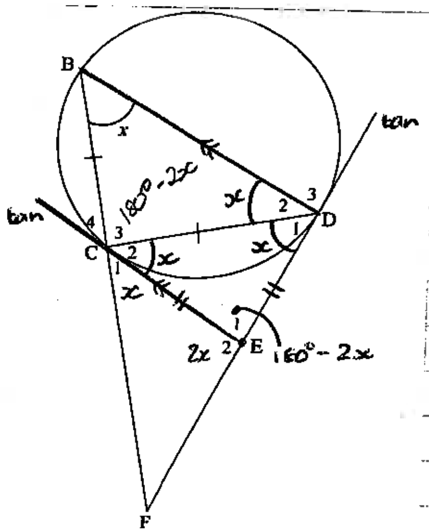
$EC.BD = CD.CB$ ✓

but $CD = CB$ ✓ given

$EC.BD = CD.CD$

$EC.BD = CD^2$

10.22.



$\hat{C}_3 = 180 - 2x$ — sum $\angle$'s in $\Delta = 180°$
$\hat{C}_1 = x$ — $\angle$'s on str line $= 180°$
$\hat{E}_2 = 2x$ — $\angle$'s str line $= 180°$ ✓

In Δ's $C_{1+2}FD_1$, E_2FC_1

1. $\hat{C}_1 + \hat{C}_2 = \hat{E}_2$ — both $= 2x$
2. $\hat{D}_1 = \hat{C}_1$ — both $= x$
3. $\hat{F} = \hat{F}$ — common

$\therefore \Delta CFD \ ||| \ \Delta EFC$ ✓ — AAA ✓

$\therefore \frac{CF}{EF} = \frac{CD}{EC}$ ✓ — $\Delta \hat{C}FD \ ||| \ \Delta \hat{E}FC$

$\frac{CF^2}{EF^2} = \frac{CD^2}{EC^2}$ ✓ — $(\ )^2$ both sides

$= \frac{CE \cdot BD}{EC \cdot EC}$ ✓ — 10.2.1

$= \frac{BD}{EC}$

$= \frac{BD}{DE}$ — 10.1.1./10.2.1. $CE = DE$

6